NAG Fortran Library Routine Document F07TWF (CTRTRI/ZTRTRI)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of **bold italicised** terms and other implementation-dependent details.

1 Purpose

F07TWF (CTRTRI/ZTRTRI) computes the inverse of a complex triangular matrix.

2 Specification

```
SUBROUTINE F07TWF(UPLO, DIAG, N, A, LDA, INFO)
ENTRY ctrti (UPLO, DIAG, N, A, LDA, INFO)
INTEGER N, LDA, INFO
complex
CHARACTER*1 UPLO, DIAG
```

The ENTRY statement enables the routine to be called by its LAPACK name.

3 Description

This routine forms the inverse of a complex triangular matrix A. Note that the inverse of an upper (lower) triangular matrix is also upper (lower) triangular.

4 References

Du Croz J J and Higham N J (1992) Stability of methods for matrix inversion *IMA J. Numer. Anal.* 12 1–19

5 Parameters

1: UPLO - CHARACTER*1

Input

On entry: indicates whether A is upper or lower triangular as follows:

```
if UPLO = 'U', A is upper triangular; if UPLO = 'L', A is lower triangular.
```

Constraint: UPLO = 'U' or 'L'.

2: DIAG - CHARACTER*1

Input

On entry: indicates whether A is a non-unit or unit triangular matrix as follows:

```
if DIAG = 'N', A is a non-unit triangular matrix;
```

if DIAG = 'U', A is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.

Constraint: DIAG = 'N' or 'U'.

3: N - INTEGER

Input

On entry: n, the order of the matrix A.

Constraint: $N \geq 0$.

4: A(LDA,*) - complex array

Input/Output

Note: the second dimension of the array A must be at least max(1, N).

On entry: the n by n triangular matrix A. If UPLO = 'U', A is upper triangular and the elements of the array below the diagonal are not referenced; if UPLO = 'L', A is lower triangular and the elements of the array above the diagonal are not referenced. If DIAG = 'U', the diagonal elements of A are not referenced, but are assumed to be 1.

On exit: A is overwritten by A^{-1} , using the same storage format as described above.

5: LDA – INTEGER

Input

On entry: the first dimension of the array A as declared in the (sub)program from which F07TWF (CTRTRI/ZTRTRI) is called.

Constraint: LDA $\geq \max(1, N)$.

6: INFO – INTEGER

Output

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO = -i, the *i*th parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO = i, a_{ii} is zero and the matrix A is singular.

7 Accuracy

The computed inverse X satisfies

$$|XA - I| < c(n)\epsilon |X| |A|,$$

where c(n) is a modest linear function of n, and ϵ is the *machine precision*.

Note that a similar bound for |AX - I| cannot be guaranteed, although it is almost always satisfied.

The computed inverse satisfies the forward error bound

$$|X - A^{-1}| \le c(n)\epsilon |A^{-1}| |A| |X|.$$

See Du Croz and Higham (1992).

8 Further Comments

The total number of real floating-point operations is approximately $\frac{4}{3}n^3$.

The real analogue of this routine is F07TJF (STRTRI/DTRTRI).

9 Example

To compute the inverse of the matrix A, where

$$A = \begin{pmatrix} 4.78 + 4.56i & 0.00 + 0.00i & 0.00 + 0.00i & 0.00 + 0.00i \\ 2.00 - 0.30i & -4.11 + 1.25i & 0.00 + 0.00i & 0.00 + 0.00i \\ 2.89 - 1.34i & 2.36 - 4.25i & 4.15 + 0.80i & 0.00 + 0.00i \\ -1.89 + 1.15i & 0.04 - 3.69i & -0.02 + 0.46i & 0.33 - 0.26i \end{pmatrix}$$

9.1 Program Text

Note: the listing of the example program presented below uses **bold italicised** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
FO7TWF Example Program Text
Mark 16 Release. NAG Copyright 1993.
.. Parameters ..
                 NIN, NOUT
INTEGER
PARAMETER
                 (NIN=5,NOUT=6)
INTEGER
                NMAX, LDA
                 (NMAX=8,LDA=NMAX)
PARAMETER
CHARACTER
                 DIAG
PARAMETER
                 (DIAG='N')
.. Local Scalars ..
           I, IFAIL, INFO, J, N
INTEGER
CHARACTER
                 UPLO
.. Local Arrays ..
complex
CHARACTER A(LDA,NMAX)
CLABS(1), RLABS(1)
.. External Subroutines ..
EXTERNAL XO4DBF, ctrtri
.. Executable Statements ..
WRITE (NOUT,*) 'F07TWF Example Program Results'
Skip heading in data file
READ (NIN, *)
READ (NIN,*) N
IF (N.LE.NMAX) THEN
   Read A from data file
   READ (NIN, *) UPLO
   IF (UPLO.EQ.'U') THEN
      READ (NIN,*) ((A(I,J),J=I,N),I=1,N)
   ELSE IF (UPLO.EQ.'L') THEN
     READ (NIN, *) ((A(I,J), J=1,I), I=1,N)
   END IF
   Compute inverse of A
   CALL ctrtri(UPLO, DIAG, N, A, LDA, INFO)
   Print inverse
   WRITE (NOUT, *)
   IFAIL = 0
   CALL XO4DBF(UPLO, DIAG, N, N, A, LDA, 'Bracketed', 'F7.4', 'Inverse',
                'Integer', RLABS, 'Integer', CLABS, 80,0, IFAIL)
END IF
STOP
END
```

9.2 Program Data

```
FO7TWF Example Program Data

4
'L'
( 4.78, 4.56)
( 2.00,-0.30) (-4.11, 1.25)
( 2.89,-1.34) ( 2.36,-4.25) ( 4.15, 0.80)
(-1.89, 1.15) ( 0.04,-3.69) (-0.02, 0.46) ( 0.33,-0.26) :End of matrix A
```

9.3 Program Results

FO7TWF Example Program Results

```
Inverse

1 2 3 4

1 (0.1095,-0.1045)
2 (0.0582,-0.0411) (-0.2227,-0.0677)
3 (0.0032, 0.1905) (0.1538,-0.2192) (0.2323,-0.0448)
4 (0.7602, 0.2814) (1.6184,-1.4346) (0.1289,-0.2250) (1.8697, 1.4731)
```